

# Math 1060 - Exam 4 Review Key

Note Title

5/2/2016

$$1. (2+7i) - (-4+2i) \\ = 2+7i+4-2i = \boxed{6+5i}$$

$$2. (4-3i)(2+8i) \\ = 8+32i-6i-\cancel{24i^2}^{+24} \\ = \boxed{32+26i}$$

$$3. \left( \frac{-5+2i}{1-3i} \right) \left( \frac{1+3i}{1+3i} \right) = \frac{-5-15i+2i+\cancel{6i^2}^{-6}}{1^2+3^2} = \frac{-11-13i}{10} = \boxed{-\frac{11}{10} - \frac{13}{10}i}$$

$$4. i^{29} = (i^2)^{14} \cdot i = (-1)^{14} \cdot i = \boxed{i} \\ \text{or } 29 \div 4 = 7 \text{ R}1 \rightarrow$$

$$5. i^{76} = (i^2)^{38} = (-1)^{38} = \boxed{1} \\ \text{or } 76 \div 4 = 19 \text{ R}0 \rightarrow$$

$$6. i^{34} = (i^2)^{17} = (-1)^{17} = \boxed{-1} \\ \text{or } 34 \div 4 = 8 \text{ R}2 \rightarrow$$

$$7. i^{23} = (i^2)^{11} \cdot i = (-1)^{11} \cdot i = \boxed{-i} \\ \text{or } 23 \div 4 = 5 \text{ R}3 \rightarrow$$

Divide  
exponent  
by 4

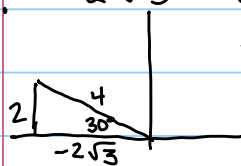
Remainder	Answer
0	1
1	i
2	-1
3	-i

$$8. (2-4i)(2+4i) = 2^2 + 4^2 = \boxed{20} \quad \text{or FOIL: } 4 + \cancel{8i} - \cancel{8i} - \cancel{16i^2}^{+16} = \boxed{20}$$

$$9. \frac{12 + \sqrt{-36}}{3} = \frac{12 + 6i}{3} = \boxed{4+2i}$$

$$10. -2\sqrt{3} + 2i$$

10.  $-2\sqrt{3} + 2i$



$30^\circ-60^\circ-90^\circ$  triangle! hyp = short leg  $\cdot 2$

$r=4 \quad \theta=150^\circ$

or  $r = \sqrt{2^2 + (2\sqrt{3})^2} = \sqrt{4+12} = 4$

$\sin \theta = \frac{2}{4} = \frac{1}{2}$

$\cos \theta = \frac{-2\sqrt{3}}{4} = -\frac{\sqrt{3}}{2} \quad \theta = 150^\circ$

$4(\cos 150^\circ + i \sin 150^\circ)$

11.  $9(\cos 240^\circ + i \sin 240^\circ)$

$= 9(-\frac{1}{2} - \frac{\sqrt{3}}{2}i) = \boxed{-\frac{9}{2} - \frac{9\sqrt{3}}{2}i}$

12.  $4 \operatorname{cis} 60^\circ \cdot 3 \operatorname{cis} 150^\circ$

$= 4 \cdot 3 \operatorname{cis} (60^\circ + 150^\circ)$

$= 12(\cos 210^\circ + i \sin 210^\circ)$

$= 12(-\frac{\sqrt{3}}{2} - \frac{1}{2}i) = \boxed{-6\sqrt{3} - 6i}$

13.  $\frac{7 \operatorname{cis}(\frac{5\pi}{6})}{2 \operatorname{cis}(\frac{\pi}{3})}$

$= \frac{7}{2} \operatorname{cis}(\frac{5\pi}{6} - \frac{\pi}{3})$

$\frac{5\pi}{6} - \frac{2\pi}{6} = \frac{3\pi}{6} = \frac{\pi}{2}$

$= \frac{7}{2}(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2})$

$= \frac{7}{2}(0 + i) = \boxed{\frac{7}{2}i}$

14.  $(2 \operatorname{cis} 225^\circ)^7$

$= 2^7 \operatorname{cis} (7 \cdot 225^\circ)$

$= 128 \operatorname{cis} 1575^\circ$

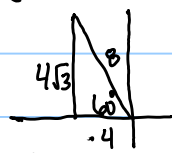
$= 128(\cos 135^\circ + i \sin 135^\circ)$

$= 128(-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i) = \boxed{-64\sqrt{2} + 64i\sqrt{2}}$

$1575^\circ - 4(360^\circ) = 135^\circ$

Keep subtracting  $360^\circ$  until you find a coterminal angle between  $0^\circ$  &  $360^\circ$ .

15.  $(-4 + 4i\sqrt{3})^4$



$30^\circ - 60^\circ - 90^\circ \Delta$

$r = 8 \quad \theta = 120^\circ$

or  $r = \sqrt{4^2 + (4\sqrt{3})^2} = \sqrt{16 + 48} = 8$

$\sin \theta = \frac{4\sqrt{3}}{8} = \frac{\sqrt{3}}{2} \quad \theta = 120^\circ$

$\cos \theta = -\frac{4}{8} = -\frac{1}{2}$

$(8 \operatorname{cis} 120^\circ)^4$

$= 8^4 \operatorname{cis} (4 \cdot 120^\circ)$

$= 4096 \operatorname{cis} 480^\circ \quad 480^\circ - 360^\circ = 120^\circ$

$= 4096 (\cos 120^\circ + i \sin 120^\circ)$

$= 4096 \left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)$

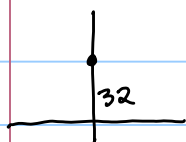
$= \boxed{-2048 + 2048i\sqrt{3}}$

16. 4<sup>th</sup> roots of  $81 \operatorname{cis} 292^\circ$

$81^{\frac{1}{4}} = 3 \quad \frac{292^\circ + 360^\circ k}{4} = 73^\circ + 90^\circ k$

$\boxed{3 \operatorname{cis} \alpha, \text{ where } \alpha = 73^\circ, 163^\circ, 253^\circ, \& 343^\circ}$

17. 5<sup>th</sup> roots of  $32i$



$r = 32 \quad \theta = 90^\circ$

5<sup>th</sup> roots of  $32 \operatorname{cis} 90^\circ$

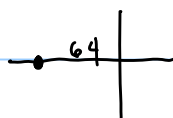
$32^{\frac{1}{5}} = 2 \quad \frac{90^\circ + 360^\circ k}{5} = 18^\circ + 72^\circ k$

$\boxed{2 \operatorname{cis} \alpha, \text{ where } \alpha = 18^\circ, 90^\circ, 162^\circ, 234^\circ, \& 306^\circ}$

18.  $x^3 + 64 = 0$

$x^3 = -64$

cube roots of  $-64$



$r = 64, \theta = 180^\circ$

cube roots of  $64 \operatorname{cis} 180^\circ$

$64^{\frac{1}{3}} = 4$

$\frac{180^\circ + 360^\circ k}{3} = 60^\circ + 120^\circ k$

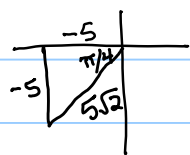
$60^\circ, 180^\circ, 300^\circ$

$4(\cos 60^\circ + i \sin 60^\circ) = 4\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) = \boxed{2 + 2i\sqrt{3}}$

$4(\cos 180^\circ + i \sin 180^\circ) = 4(-1 + 0i) = \boxed{-4}$

$4(\cos 300^\circ + i \sin 300^\circ) = 4\left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) = \boxed{2 - 2i\sqrt{3}}$

19.

 $45^\circ - 45^\circ - 90^\circ \Delta$ hyp = leg  $\sqrt{2}$ 

$r = 5\sqrt{2} \quad \theta = \frac{5\pi}{4}$

$r = \sqrt{5^2 + 5^2} = \sqrt{50} = 5\sqrt{2}$

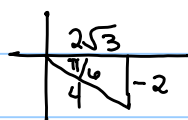
$\sin \theta = -\frac{5}{5\sqrt{2}} = -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2} \quad \cos \theta = -\frac{5}{5\sqrt{2}} = -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2} \quad \theta = \frac{5\pi}{4}$

$\boxed{(5\sqrt{2}, \frac{5\pi}{4})}$

20.  $(-4, 0)$   $r = 4 \quad \theta = \pi$

$\boxed{(4, \pi)}$

21.  $(2\sqrt{3}, -2)$



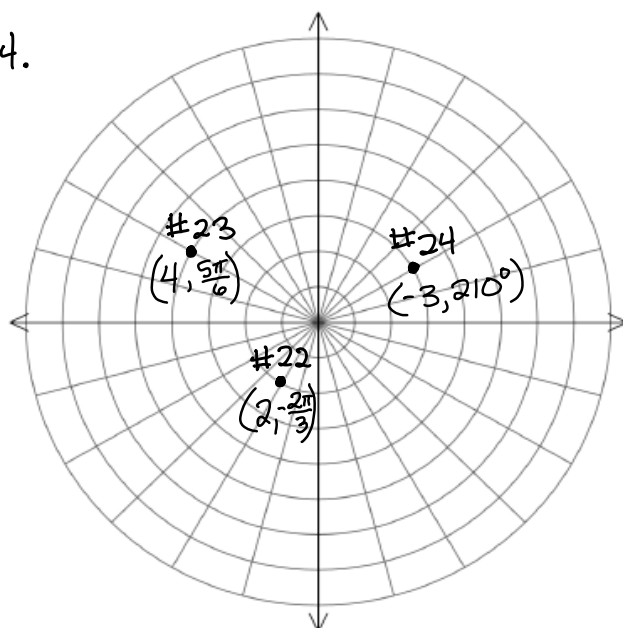
or  $r = \sqrt{(2\sqrt{3})^2 + 2^2} = \sqrt{12 + 4} = 4$

$\sin \theta = -\frac{2}{4} = -\frac{1}{2} \quad \cos \theta = \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2} \quad \theta = \frac{11\pi}{6}$

$\boxed{(4, \frac{11\pi}{6})}$

$30^\circ - 60^\circ - 90^\circ \Delta$   
 $r = 4 \quad \theta = \frac{11\pi}{6}$

22.-24.



25.  $(-2, \frac{3\pi}{4})$

$x = -2 \cos(\frac{3\pi}{4}) = -2(-\frac{\sqrt{2}}{2}) = \sqrt{2}$

$y = -2 \sin(\frac{3\pi}{4}) = -2(\frac{\sqrt{2}}{2}) = -\sqrt{2}$

$\boxed{(\sqrt{2}, -\sqrt{2})}$

26.  $(3, -\frac{1}{2}\pi)$

$$x = 3 \cos(-\frac{\pi}{2}) = 3(0) = 0$$

$$y = 3 \sin(-\frac{\pi}{2}) = 3(-1) = -3$$

$$\boxed{(0, -3)}$$

27.  $(-4, \frac{4\pi}{3})$

$$x = -4 \cos \frac{4\pi}{3} = -4(-\frac{1}{2}) = 2$$

$$y = -4 \sin \frac{4\pi}{3} = -4(-\frac{\sqrt{3}}{2}) = 2\sqrt{3}$$

$$\boxed{(2, 2\sqrt{3})}$$

28.  $r = 7$

See graph paper

No matter what  $\theta$  is,  $r$  is 7

29.  $r = -6 \sin \theta$  see graph paper

$\theta$	$0^\circ$	$15^\circ$	$30^\circ$	$45^\circ$	$60^\circ$	$75^\circ$	$90^\circ$
$r$	0	-1.6	-3	-4.2	-5.2	-5.8	-6

symmetric around y-axis

30.  $r = 3 - 3 \cos \theta$  see graph paper

$\theta$	$0^\circ$	$15^\circ$	$30^\circ$	$45^\circ$	$60^\circ$	$75^\circ$	$90^\circ$
$r$	0	.1	.4	.9	1.5	2.2	3

$\theta$	$105^\circ$	$120^\circ$	$135^\circ$	$150^\circ$	$165^\circ$	$180^\circ$
$r$	3.8	4.5	5.1	5.6	5.9	6

symmetric  
around x-axis

31.  $r^2 = 4 \sin(2\theta)$  see graph paper

$\theta$	$0^\circ$	$15^\circ$	$30^\circ$	$45^\circ$	$60^\circ$	$75^\circ$	$90^\circ$
$r^2$	0	2	$2\sqrt{3}$	4	$2\sqrt{3}$	2	0

$r$	0	$\pm 1.4$	$\pm 1.9$	$\pm 2$	$\pm 1.9$	$\pm 1.4$	0
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32.  $r = 3 + 6 \sin \theta$  See graph paper

$\theta$	$0^\circ$	$15^\circ$	$30^\circ$	$45^\circ$	$60^\circ$	$75^\circ$	$90^\circ$
$r$	3	4.6	6	7.2	8.2	8.8	9

Symmetric around y-axis

$\theta$	$-15^\circ$	$-30^\circ$	$-45^\circ$	$-60^\circ$	$-75^\circ$	$-90^\circ$
$r$	1.4	0	-1.2	-2.2	-2.8	-3

33.  $r = 4 \sin(3\theta)$  See graph paper

$\theta$	$0^\circ$	$15^\circ$	$30^\circ$	$45^\circ$	$60^\circ$	$75^\circ$	$90^\circ$	$105^\circ$	$120^\circ$
$r$	0	2.8	4	2.8	0	-2.8	-4	-2.8	0

$\theta$	$135^\circ$	$150^\circ$	$165^\circ$	$180^\circ$
$r$	2.8	4	2.8	0

34.  $r = 5 - 2 \sin \theta$  See graph paper

$\theta$	$0^\circ$	$15^\circ$	$30^\circ$	$45^\circ$	$60^\circ$	$75^\circ$	$90^\circ$
$r$	5	4.5	4	3.6	3.3	3.1	3

Symmetric around y-axis

$\theta$	$-15^\circ$	$-30^\circ$	$-45^\circ$	$-60^\circ$	$-75^\circ$	$-90^\circ$
$r$	5.5	6	6.4	6.7	6.9	7

35.  $r = 5 \cos(4\theta)$  See graph paper

$\theta$	$0^\circ$	$22.5^\circ$	$45^\circ$	$67.5^\circ$	$90^\circ$	$112.5^\circ$	$135^\circ$
$r$	5	0	-5	0	5	0	-5

$\theta$	$157.5^\circ$	$180^\circ$	$202.5^\circ$	$225^\circ$	$247.5^\circ$	$270^\circ$
$r$	0	5	0	-5	0	5

36.  $r = 7$

$r^2 = 49$  (square both sides)

$x^2 + y^2 = 49$

37.  $r = 5 \cos \theta$

$r^2 = 5r \cos \theta$  (multiply both sides by  $r$ )

$\boxed{x^2 + y^2 = 5x}$

38.  $r = 7 \csc \theta$

$r = \frac{7}{\sin \theta}$

$(\csc \theta = \frac{1}{\sin \theta})$

$r \sin \theta = 7$

$\boxed{y = 7}$

39.  $x^2 + y^2 = 64$

$r^2 = 64$

$\boxed{r = 8}$

40.  $x = 5$

$r \cos \theta = 5$

$r = \frac{5}{\cos \theta}$

$\boxed{r = 5 \sec \theta}$

41.  $x^2 + y^2 + 5y = 0$

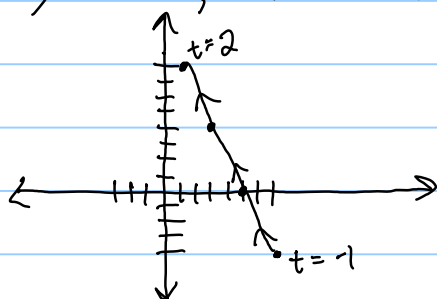
$r^2 + 5r \sin \theta = 0$

$r^2 = -5r \sin \theta$

$\boxed{r = -5 \sin \theta}$  (Divide both sides by  $r$ )

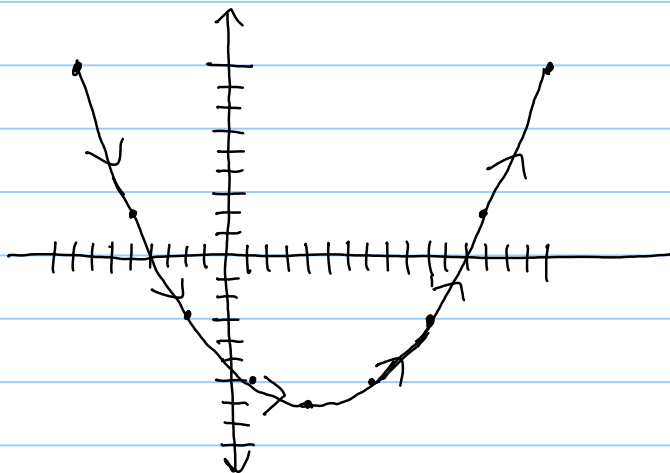
42.  $x = -2t + 5, y = 4t; -1 \leq t \leq 2$

$t$	$x$	$y$
-1	7	-4
0	5	0
1	3	4
2	1	8



43.  $x = 3t + 4$ ,  $y = t^2 - 7$ ;  $-4 \leq t \leq 4$

$t$	$x$	$y$
-4	-8	9
-3	-5	2
-2	-2	-3
-1	1	-6
0	4	-7
1	7	-6
2	10	-3
3	13	2
4	16	9



44.  $x = 8t$   $y = 4t + 9$   
 $t = \frac{x}{8}$   $y = 4\left(\frac{x}{8}\right) + 9$   
 $y = \frac{1}{2}x + 9$  line

45.  $x = t + 5$   $y = t^2 + 3$   
 $t = x - 5$   $y = (x - 5)^2 + 3$   
 $y = x^2 - 10x + 25 + 3$   
 $y = x^2 - 10x + 28$  parabola

46.  $x = 4 \cos \theta$   $y = \sin \theta$   
 $\frac{x}{4} = \cos \theta$   
 $\frac{x^2}{16} = \cos^2 \theta$   $y^2 = \sin^2 \theta$   
 $\frac{x^2}{16} + y^2 = \cos^2 \theta + \sin^2 \theta$   
 $\frac{x^2}{16} + y^2 = 1$  ellipse



$$47. \quad t=0: (1, -2)$$

$$x = m_1 t + b_1$$

$$1 = m_1(0) + b_1$$

$$b_1 = 1$$

$$13 = 3m_1 + b_1$$

$$13 = 3m_1 + 1$$

$$12 = 3m_1$$

$$m_1 = 4$$

$$\boxed{x = 4t + 1}$$

$$t=3: (13, 1)$$

$$y = m_2 t + b_2$$

$$-2 = m_2(0) + b_2$$

$$b_2 = -2$$

$$1 = 3m_2 + b_2$$

$$1 = 3m_2 - 2$$

$$3 = 3m_2$$

$$m_2 = 1$$

$$\boxed{y = t - 2}$$

$$48. \quad t=3: (2, 5)$$

$$x = m_1 t + b_1$$

$$-1(2 = 3m_1 + b_1) \leftarrow \begin{matrix} t=3 \\ x=2 \end{matrix}$$

$$-10 = 7m_1 + b_1 \leftarrow \begin{matrix} t=7 \\ x=-10 \end{matrix}$$

$$\underline{-2 = -3m_1 - b_1}$$

$$-12 = 4m_1$$

$$m_1 = -3$$

$$2 = 3(-3) + b_1$$

$$2 = -9 + b_1$$

$$b_1 = 11$$

$$\boxed{x = -3t + 11}$$

$$t=7: (-10, 13)$$

$$y = m_2 t + b_2$$

$$-1(5 = 3m_2 + b_2) \leftarrow \begin{matrix} t=3 \\ y=5 \end{matrix}$$

$$13 = 7m_2 + b_2 \leftarrow \begin{matrix} t=7 \\ y=13 \end{matrix}$$

$$\underline{-5 = -3m_2 - b_2}$$

$$8 = 4m_2$$

$$m_2 = 2$$

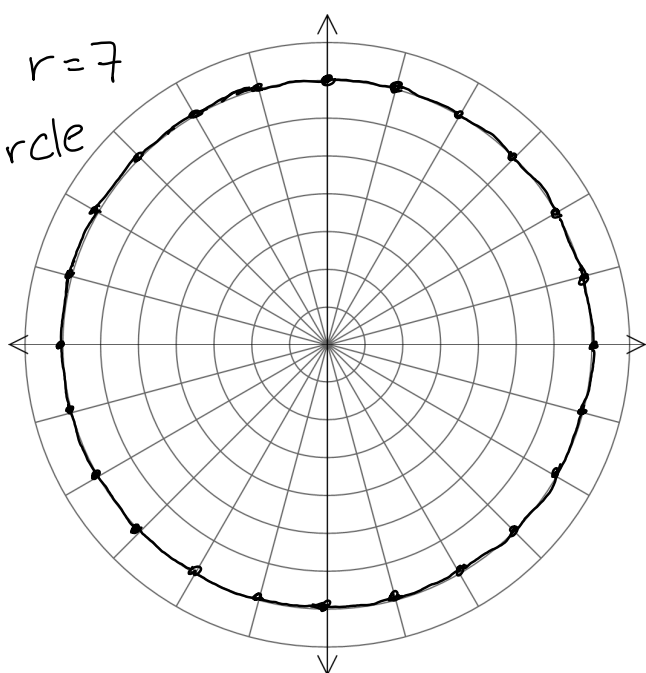
$$5 = 3(2) + b_2$$

$$5 = 6 + b_2$$

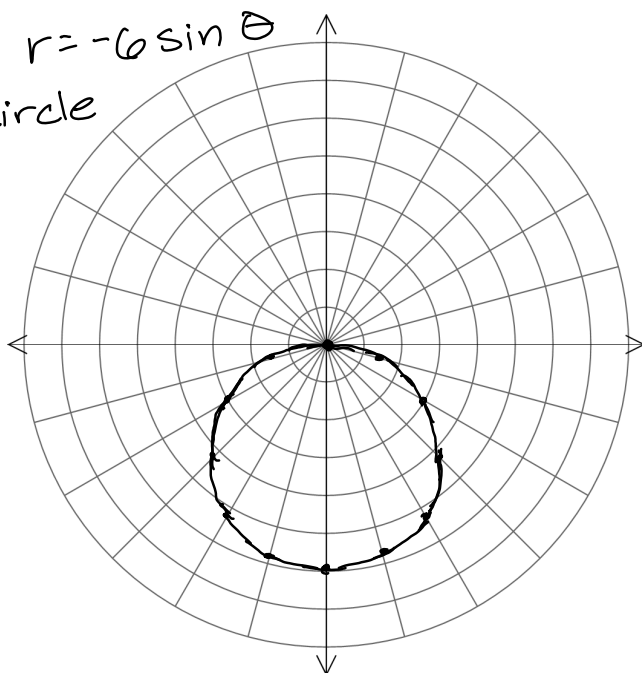
$$b_2 = -1$$

$$\boxed{y = 2t - 1}$$

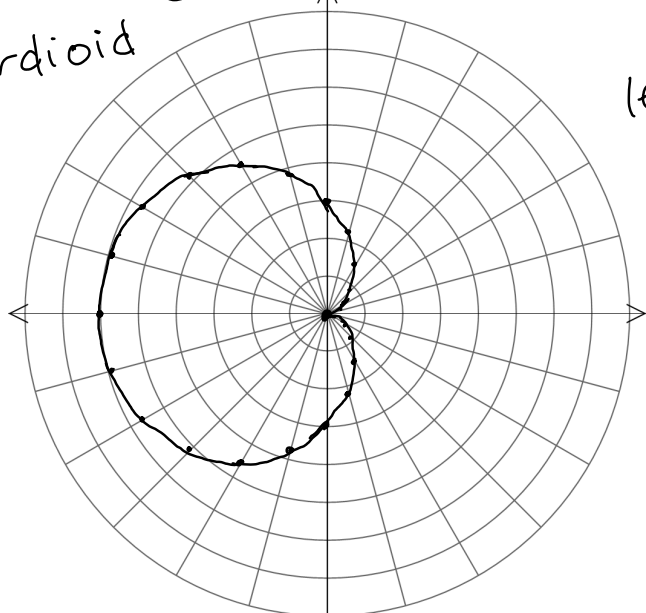
28.  $r=7$   
circle



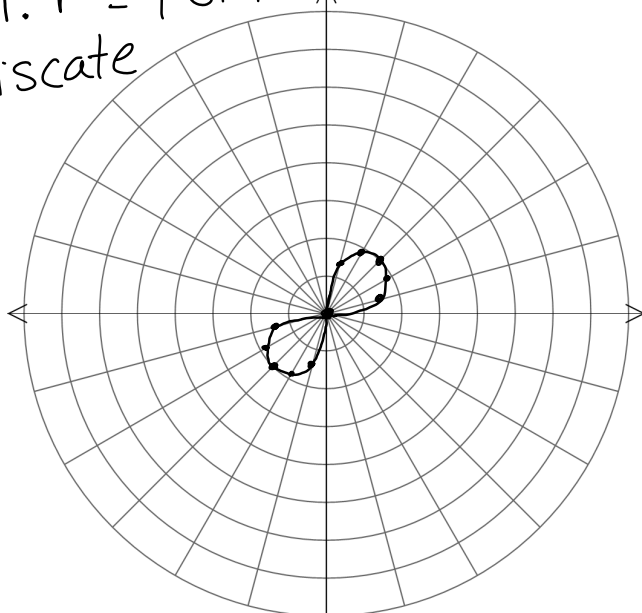
29.  $r=-6\sin\theta$   
circle



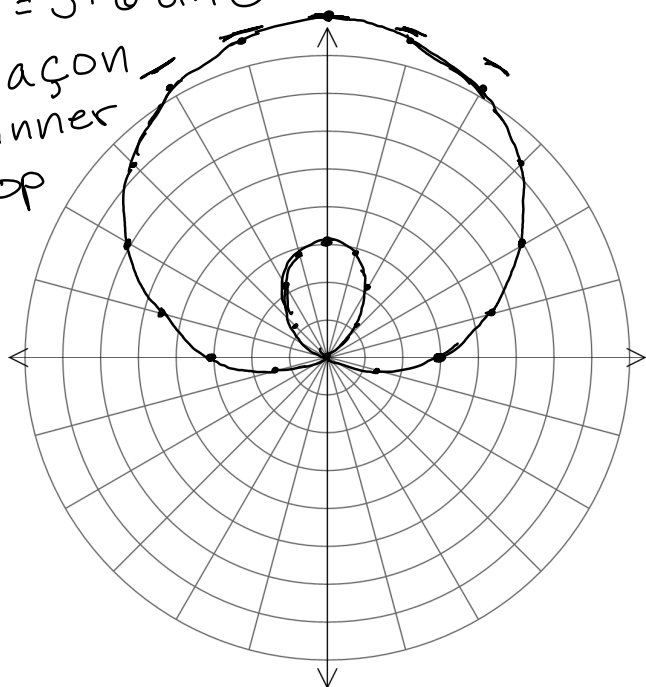
30.  $r=3-3\cos\theta$   
cardioid



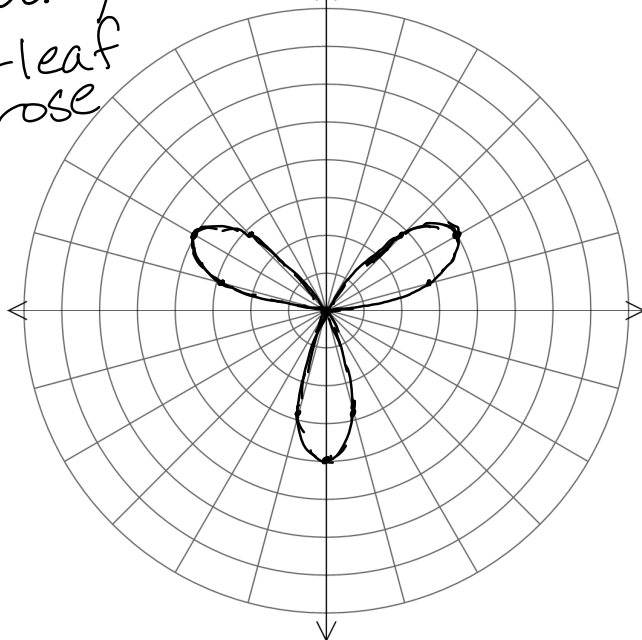
31.  $r^2=4\sin(2\theta)$   
lemniscate



32.  $r=3+6\sin\theta$   
limaçon  
w/ inner  
loop

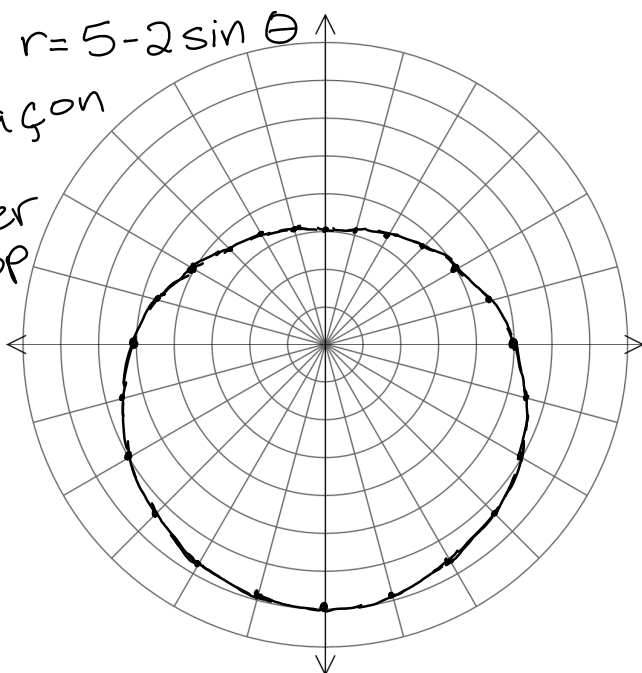


33.  $y=4\sin(3\theta)$   
3-leaf  
rose



34.  $r = 5 - 2 \sin \theta$

limaçon  
w/o  
inner  
loop



35.  $r = 5 \cos(4\theta)$

8-leaf  
rose

